

# THE LAGRANGIAN LOOP REPRESENTATION OF LATTICE $U(1)$ GAUGE THEORY

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## Abstract

It is showed how the Hamiltonian lattice *loop representation* can be cast straightforwardly in the Lagrangian formalism. The procedure is general and here we present the simplest case: pure compact QED. This connection has been shaded by the non canonical character of the algebra of the fundamental loop operators. The loops represent tubes of electric flux and can be considered the dual objects to the Nielsen-Olesen strings supported by the Higgs broken phase. The lattice loop classical action corresponding to the Villain form is proportional to the quadratic area of the loop world sheets and thus it is similar to the Nambu string action. This loop action is used in a Monte Carlo simulation and its appealing features are discussed.

# 1 Introduction

A unified quantum theory which describes the gauge fields and the gravitation is one of the main goals pursued by the physicists for long time. A good candidate for accomplishing this comprehensive framework is the *loop representation*. This loop approach was introduced in the early eighties by Gambini and Trias [1],[2] as a Hamiltonian representation of gauge theories in terms of their natural physical excitations: the loops. The original aim of this general analytical Hamiltonian approach for gauge theories was to avoid the redundancy introduced by the gauge symmetry working directly in the space of physical states. However, soon it was realized that the loop formalism goes far beyond of a simple gauge invariant description. The introduction by Ashtekar [3] of a new set of variables that cast general relativity in the same language as gauge theories allowed to apply loop techniques as a natural non-perturbative description of Einstein's theory. Furthermore, the loop representation appeared as the most appealing application of the loop techniques to this problem [4],[5].

The Hamiltonian techniques for gauge theories have been developed during the last decade and they provide interesting results for several lattice models [6]-[9]. On the other hand a Lagrangian approach in terms of loops has been elusive, due mainly to the non-canonical character of the loop algebra. This feature forbids the possibility of performing a Legendre transformation as a straightforward way to obtain the Lagrangian from the Hamiltonian.

In the case of non-Abelian gauge theory a major problem has been whether we can write a reasonably simple Lagrangian in terms of "electric vector potentials" [10]. A Lagrangian loop formulation will give rise to new computation techniques providing a useful complement to the Hamiltonian loop studies.

Recently, it was proposed a tentative classical action in terms of loop variables for the  $U(1)$  gauge theory [11]. Shortly afterwards, we proved that the lattice version of this action is equivalent to Villain form for  $D=2+1$  dimensions but is slightly different for  $D=3+1$  dimensions [12]. In fact, this action written in terms of variables directly attached to spatial loops seems to fail in describing all the dynamical degrees of freedom for  $D=4$ .

This paper is organized as follows. In section 2 we show how the loops, originally thought up within the Hamiltonian formalism, can be introduced in a natural way in the lattice Lagrangian theory. We follow a different approach to that of reference [12]: we show how the electric loops can be traced in the statistical lattice formulation of  $4d$   $U(1)$  theory giving rise to an expression of the partition function as a sum of integer closed surfaces. We interpret these surfaces as the world sheets of the electric loops. We discuss the connection of this classical loop action with the Nambu string action. A clear analogy is patent between the gauge theory in the loop representation and the bosonic and fermionic strings, as it was previously suspected [13], [14] but never (as far as we know) demonstrated explicitly. The parallelism of the loop representation with the topological representation of the broken Higgs phase in terms of Nielsen-Olesen strings [15] is also pointed out. In section 3 we use the loop action equivalent to the Villain form for performing a Monte

Carlo simulation. It turns out that this action is the same as the  $\gamma \rightarrow \infty$  limit for the non-compact Abelian-Higgs theory [16] ( $\gamma$  : Higgs coupling constant).

## 2 The Lagrangian loop Representation

The loop based approach of ref.[1] describes the quantum electrodynamics in terms of the gauge invariant holonomy (Wilson loop)

$$\hat{W}(\gamma) = \exp[ie \oint_{\gamma} A_a(y) dy^a], \quad (1)$$

and the conjugate electric field  $\hat{E}^a(x)$ . They obey the commutation relations

$$[\hat{E}^a(x), \hat{W}(\gamma)] = e \int_{\gamma} \delta(x - y) dy^a \hat{W}(\gamma). \quad (2)$$

These operators act on a state space of abelian loops  $\psi(\gamma)$  that may be expressed in terms of the transform

$$\psi(\gamma) = \int d_{\mu}[A] \langle \gamma | A \rangle \langle A | \psi \rangle = \int d_{\mu}[A] \psi[A] \exp[-ie \oint_{\gamma} A_a dy^a]. \quad (3)$$

This loop representation has many appealing features: In first place, it allows to do away with the first class constraints of gauge theories. That is, the Gauss law is automatically satisfied. In second place, the formalism only involves gauge invariant objects. Finally, all the gauge invariant operators have a transparent geometrical meaning when they are realized in the loop space.

When this loop representation is implemented in the lattice it offers a gauge invariant description of physical states in terms of kets  $|C\rangle = \hat{W}(C) |0\rangle$ , where  $C$  labels a closed path in the *spatial* lattice. Eq.(2) becomes

$$[\hat{E}_l, \hat{W}(C)] = N_l(C) \hat{W}(C), \quad (4)$$

where  $l$  denotes the links of the lattice,  $\hat{E}(l)$  the lattice electric field operator,  $\hat{W}(C) = \prod_{l \in C} \hat{U}(l)$  and  $N_l(C)$  is the number of times that the link  $l$  appears in the closed path  $C$ .

In this loop representation, the Wilson loop acts as the loop creation operator:

$$\hat{W}(C') |C\rangle = |C' \cdot C\rangle. \quad (5)$$

The physical meaning of an abelian loop may be deduced from (4) and (5), in fact

$$\hat{E}_l |C\rangle = N_l(C) |C\rangle, \quad (6)$$

which implies that  $|C\rangle$  is an eigenstate of the electric field. The corresponding eigenvalue is different from zero if the link  $l$  belongs to  $C$ . Thus  $C$  represents a confined line of electric flux.

In order to cast the loop representation in Lagrangian form it is convenient to use the language of differential forms on the lattice of ref.[17]. Besides the great simplifications to which this formalism lead its advantages consists in the general character of the expressions obtained. That is, most of the transformations are independent on the space-time dimension or on the rank of the fields. So let us summarize the main concepts and some useful results of the formalism of differential forms on the lattice.

A k-form is a function defined on the k-cells of the lattice (k=0 sites, k=1 links, k=2 plaquettes, etc.) over an abelian group which shall be  $\mathbf{R}$ ,  $\mathbf{Z}$ , or  $U(1)$ =reals module  $2\pi$ .

Integer forms can be considered geometrical objects on the lattice. For instance, a 1-form represents a path and the integer value on a link is the number of times that the path traverses this link.

Let us introduce  $\nabla$  is the co-border operator which maps k-forms onto (k+1)-forms. It is the gradient operator when acting on scalar functions (0-forms) and it is the rotational on vector functions (1-forms). We shall consider the scalar product of p-forms defined  $\langle \alpha | \beta \rangle = \sum_{c_k} \alpha(c) \beta(c)$  where the sum runs over the k-cells of the lattice. Under this product the  $\nabla$  operator is adjoint to the border operator  $\partial$  which maps k-forms onto (k-1)-forms and which corresponds to minus times the usual divergence operator. That is,

$$\langle \alpha | \nabla \beta \rangle = \langle \partial \alpha | \beta \rangle, \quad (7)$$

$$\langle \nabla \alpha | \beta \rangle = \langle \alpha | \partial \beta \rangle. \quad (8)$$

The co-border  $\nabla$  and border  $\partial$  operators verify

$$\nabla^2 = 0, \quad \partial^2 = 0. \quad (9)$$

The Laplace-Beltrami operator is defined by

$$\square = \nabla \partial + \partial \nabla. \quad (10)$$

It is a symmetric linear operator which commutes with  $\nabla$  and  $\partial$ , and differs only by a minus sign of the current Laplacian  $\Delta_\mu \Delta_\mu$ .

From Eq.(10) is easy to show the Hodge-identity:

$$1 = \partial \square^{-1} \nabla + \nabla \square^{-1} \partial. \quad (11)$$

A useful tool to consider is the *duality transformation* which maps bijectively k-forms over (D-k)-forms. We denote by  $*p_{c_{D-k}}$  the dual form of the  $p_{c_k}$  form. For example, for  $D = 2$ , to plaquettes there correspond sites of the dual lattice, i.e. those vertices obtained from the original ones by a translation of vector  $(a/2, a/2)$ .

Under duality the border and co-border operators interchange:

$$\partial = *\nabla*. \quad (12)$$

After this digression about differential forms on the lattice let us consider the generating functional for the Wilson U(1) lattice action:

$$Z_W = \int_{-\pi}^{\pi} (d\theta_l) \exp(-\frac{\beta}{2} \sum_p \cos \theta_p), \quad (13)$$

where the subscripts  $l$  and  $p$  stand for the lattice links and plaquettes respectively.

Fourier expanding the  $\exp[\cos \theta]$  we get

$$Z_W = \int_{-\pi}^{\pi} (d\theta_l) \prod_p \sum_{n_p} I_{n_p}(\beta) e^{i n_p \theta_p}, \quad (14)$$

which can be written, using the language of differential forms as

$$Z_W = \sum_{\{n_p\}} \int_{-\pi}^{\pi} (d\theta_l) \exp(\sum_p \ln I_{n_p}(\beta)) e^{i \langle n, \nabla \theta_l \rangle}. \quad (15)$$

In the above expression,  $\theta_l$  is a real periodic 1-form, that is, a real number  $\theta \in [-\pi, \pi]$  defined on each link of the lattice;  $\nabla$  is the co-border operator;  $n_p$  are integer 2-forms, defined at the lattice plaquettes.

By eq. (8) and integrating over  $\theta_l$  we obtain a  $\delta(\partial n_p)$ . Then,

$$Z_W \propto \sum_{\{n_p; \partial n_p=0\}} \exp(\sum_p \ln I_{n_p}(\beta)), \quad (16)$$

the constraint  $\partial n_p = 0$  means that the sum is restricted to *closed* 2-forms. Thus, the sum runs over collections of plaquettes constituting closed surfaces. This expression was obtained by Savit [18] as an intermediate step towards the *dual* representation.

An alternative and more easy to handle lattice action than the Wilson form is the Villain form. The partition function of that form is given by

$$Z_V = \int (d\theta) \sum_s \exp(-\frac{\beta_V}{2} || \nabla \theta - 2\pi s ||^2), \quad (17)$$

where  $|| \dots ||^2 = \langle \dots, \dots \rangle$ . If we use the Poisson summation formula

$$\sum_s f(s) = \sum_n \int_{-\infty}^{\infty} d\phi f(\phi) e^{2\pi i \phi n}$$

and we integrate the continuum  $\phi$  variables we get

$$Z_V = (2\pi\beta_V)^{-N_p/2} \int (d\theta) \sum_n \exp(-\frac{1}{2\beta_V} \langle n, n \rangle + i \langle n, \nabla \theta \rangle), \quad (18)$$

where  $N_p$  is the number of plaquettes of the lattice. Again, we can use the equality:  $\langle n, \nabla \theta \rangle = \langle \partial n, \theta \rangle$  and integrating over  $\theta$  we obtain a  $\delta(\partial n)$ . Then,

$$Z_V = (2\pi\beta_V)^{-N_p/2} \sum_{\{n; \partial n=0\}} \exp(-\frac{1}{2\beta_V} \langle n, n \rangle), \quad (19)$$

where  $n$  are integer 2-forms. Eq. (19) is obtained from Eq.(16) in the  $\beta \rightarrow \infty$  limit.

If we consider the intersection of one of such surfaces with a  $t = \text{constant}$  plane we get a loop  $C_t$ . It is easy to show that the creation operator of this loop is just the creation operator of the loop representation, namely the Wilson loop operator. Repeating the steps from Eq.(17) to Eq.(19) we get for  $\langle \hat{W}(C_t) \rangle$

$$\langle W(C_t) \rangle = \frac{1}{Z} (2\pi\beta_V)^{-N_p/2} \sum_{\substack{n \\ (\partial n = C_t)}} \exp\left(-\frac{1}{2\beta_V} \langle n, n \rangle\right). \quad (20)$$

This is a sum over all closed world sheets and over all world sheets spanned on the loop  $C_t$ . In other words, we have arrived to an expression of the partition function of compact electrodynamics in terms of the world sheets of loops: the *loop* (Lagrangian) representation.

There are other equivalent representations which can be obtained from the Villain form. First, we have the *dual* representation [18] obtained essentially by using the Poisson identity and then performing a duality transformation. Actually, the loop representation for the compact U(1) gauge model is reached following this procedure but stopping before the duality transformation. Second, for any lattice theory with Abelian compact variables, the ‘*topological*’ or *BKT* (for Berezinskii-Kosterlitz-Thouless) representation [19] via the ‘*Banks – Kogut – Myerson*’ transformation [20]. The *BKT* expression for the partition function of a lattice theory with Abelian compact variables is given by

$$Z_V \propto \sum_{\substack{*t \\ (\partial *t = 0)}} \exp\left(-\frac{2\pi^2}{g^2} \langle *t, \hat{\Delta} *t \rangle\right), \quad (21)$$

i.e. a sum over closed  $(D - k - 2)$  topological forms  $*t$  attached to the cells  $c_{(D-k-2)}$  of the dual lattice and where  $\hat{\Delta}$  represents the propagator operator. In the case of compact electrodynamics,  $*t \equiv *m$  i.e. the topological objects are monopoles (particles for  $D=2+1$  and loops for  $D=3+1$ ) and  $\hat{\Delta} \equiv \frac{1}{\square}$ .

Returning to the loop representation of the partition function Eq.(19) we can observe that the loop action is proportional to the *quadratic area*  $A_2$ :

$$S_V = -\frac{1}{\beta_V} A_2 = -\frac{1}{\beta_V} \sum_{p \in S} n_p^2 = -\frac{1}{\beta_V} \langle n, n \rangle, \quad (22)$$

i.e. the sum of the squares of the multiplicities  $s_p$  of plaquettes which constitute the loop’s world sheet  $S$ . It is interesting to note the similarity of this action with the continuous Nambu action or its lattice version, the Weingarten action [21] which are proportional to the area swept out by the bosonic string <sup>1</sup>.

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<sup>1</sup>The relation between the surfaces of the Wilson action and those of Weingarten action has been analyzed by Kazakov et al in ref.[22].

On the other hand, we know that in the continuum the classical action of topological string-like solitons, namely Nielsen-Olesen vortices, reduces to the Nambu action in the strong coupling limit [15]. The Nielsen-Olesen strings are static solutions of the Higgs Abelian model. A patent analogy is observed when we compare the *loop*-representation of lattice compact pure *QED* and the *BKT*-representation of lattice *Higgs* non-compact *QED*. (We compare with the non-compact instead of the compact version because this last, in addition to Nielsen-Olesen strings, also has Dirac monopoles as topological solutions and then we have to consider open as much as closed world sheets. In ref. [23] we considered the *Higgs compact QED* which exhibits duality between both representations).

In the case of *Higgs* non-compact *QED* model ( a non compact gauge field  $A_\mu$  coupled to a scalar field  $\Phi = |\Phi|e^{i\phi}$ ) we have  $t* \equiv \sigma*$ , where  $\sigma*$  represents a 2-form which corresponds to the world sheet of the topological objects namely the Nielsen-Olesen strings [24] and  $\hat{\Delta} \equiv \frac{1}{\square + M^2}$  ( $M$  is the mass acquired by the gauge field due to the Higgs mechanism). Thus, both models consist in a sum over closed surfaces which are the world sheets of closed electric strings (loops) and closed magnetic strings (closed Nielsen-Olesen vortices) respectively. The corresponding lattice actions are essentially the quadratic area of the world sheets in both cases. Moreover, the creation operator of both loops and N.O. strings is essentially the same: the Wilson loop operator [1] [24].

It is also possible to regard the connection between this two models from a different point of view : the Villain form of  $U(1)$  is the Higgs coupling  $\rightarrow \infty$  limit of *Higgs non-compact QED*. The standard action of *Higgs non-compact QED* is given by

$$S = -\frac{\beta}{2} \sum_p \theta_p^2 + \gamma \sum_l \bar{\phi}_x U(l) \phi_{x+l}. \quad (23)$$

It was pointed out in reference [16] that in the limit  $\gamma \rightarrow \infty$

$$U(l) = e^{i\theta_l} \rightarrow 1,$$

which implies for the angular variables

$$\theta_l = 2\pi n_l,$$

and so, the action (23) becomes

$$S_\infty = -\frac{\beta}{2} \sum_{p \in \mathcal{S}} (2\pi n_p)^2 = -2\pi^2 \beta < n, n >, \quad (24)$$

where  $n_p = \nabla n_l$ . We note that this is just the Villain action of gauge  $U(1)$  model, Eq.(22) but with  $2\pi^2\beta$  instead of  $\frac{1}{\beta_V}$ .

### 3 Numerical Computations

Here we shall present the results of numerical simulations carried out for the loop action (22) corresponding to Villain form of  $U(1)$  model. In fact, we have simulated the *dual* action in terms of the dual integer variables  $*n_p$

$$S_d = -\frac{1}{\beta_V} \sum_p (*n_p)^2 = -\frac{1}{\beta_V} \sum_p (\nabla * n_l)^2 = -\frac{1}{\beta_V} \sum_p \left( \sum_{l \in p} *n_l \right)^2, \quad (25)$$

where  $*n_l$  are integer 1-forms attached to links of the dual lattice and the integer 2-forms  $*n_p$  correspond to their lattice curl, i.e.  $\partial n_p = 0$  means that  $n_p = \partial n_c$  where  $n_c$  are integer 3-forms attached to the elementary cubes and according to (12)  $*n_p = \nabla * (n_c) = \nabla * n_l$ . Note that this action implies the assignment of unbounded integer variables to the links of the lattice and the action is defined through the square of the ordered sum of the integers of an elementary square of the lattice. We have implemented a Metropolis algorithm fixing the acceptance ratio, as it is usual in random surfaces analysis.

We have studied this model for different lattice sizes, imposing periodic boundary conditions. Simple thermal cycles showed the presence of a phase transition in the neighborhood of  $\beta_V = 0.639$ . To study the order of this transitions we have analyzed the energy histogram and we have checked for the presence of tunneling between the phases. We have not applied any reweighting extrapolation technique, only a direct observation of the histograms.

In Fig. 1a we present the histogram of the plaquette energy density including 80.000 measures, after discarding 40.000 thermalizing iterations, for a  $12^4$  lattice just at the Villain transition point  $\beta_V = 0.639$ . One can observe clearly a two-peak structure.

In Fig. 1b we present the time evolution of the total internal energy during the simulation. Each point is the average over 100 consecutive iterations. This analysis shows clearly the presence of "tunneling" between the phases.

Our numerical results, using this loop action equivalent to Villain form and imposing periodic boundary conditions (*PBC*), exhibit a first order phase transition signal. This is in agreement with the standard lattice numerical simulations of QED using the Wilson action (again using the standard *PBC*) [25],[26]. Nevertheless, some differences between the simulations using Wilson or Villain actions arise. In particular, we have not observed here the strange persistence of the phases -and the absence of tunneling- seen in the simulations performed with the Wilson action. Remember that according [27]-[29] the first order nature of this phase transition seems to be a spurious effect produced by the non-trivial topology that periodic boundary conditions bear with.

Typical loop configurations, obtained by intersecting the lattice with  $x_4 = \text{constant}$  planes, are showed in Fig.2 for two values of the coupling constant, one to the left (strong coupling) and one to the right (weak coupling) of the transition point. The difference between the two typical configurations is patent. These configurations are obtained by taking the last measure after thermalization with 20.000 iterations and they represent all the plaquettes with integer value  $n_p \neq 0$ . In order to obtain this plaquettes we proceed



as follows: first, we stored the  $*n_p \neq 0$  of the dual plaquettes and then we performed a duality transformation to get the corresponding non zero  $n_p$ .

In TABLE 1 we present the spectra of plaquettes  $\{n_p\}$  (in all the lattice, not only in a particular cut  $x_4 = t$ ) for different values of the  $\beta_V$  coupling. Comparing the  $\{n_p\}$  at  $\beta_V=0.637$  and  $\beta_V=0.641$  i.e. just before and after the critical Villain coupling  $\beta_c=0.639$  one can observe an abrupt increment of non zero plaquettes.

TABLE 1.

Plaquette Configurations									
$\beta_V$	$n = -4$	$n = -3$	$n = -2$	$n = -1$	$n = 0$	$n = 1$	$n = 2$	$n = 3$	$n = 4$
0.200	0	0	0	15	124386	15	0	0	0
0.500	0	0	2	1657	121099	1659	0	0	0
0.555	0	0	2	3661	117093	3655	5	0	0
0.637	0	0	70	12208	99850	12228	60	0	0
0.641	0	0	115	15104	93972	15116	109	0	0
0.714	0	0	299	19573	84639	19639	266	0	0
1.000	1	12	1292	25787	70239	25776	1300	9	0
2.000	12	569	6695	30141	49474	30337	6630	543	15

Incidentally, we want to remark that the apparent first order behaviour of the *Higgs non-compact QED* in the  $\gamma \rightarrow \infty$  limit found in ref. [16] can be clarified under this new perspective. In that limit it turns out a sort of "compactification" which transforms the original model in the pure *compact QED* (Villain form) we have studied here.

## 4 Conclusions

As it was mentioned, the loop space provides a common scenario for a non-local description of gauge theories and quantum gravity. Up to now, the loop approach was exclusively a Hamiltonian formalism and no lagrangian counterpart was available. A classical action for the Yang-Mills theory in terms of loop variables would be very valuable in its own right because they are the natural candidates to describe the theory in a confining phase. In addition, it may be useful to obtain semiclassical approximations to gauge theories or to general relativity in terms of Ashtekar's variables. Here we present some small steps in this direction which continue those ones of ref. [11] and [12].

In relation with the considered case of lattice Abelian gauge theory, we observed the known analogy between the confining phase of lattice electrodynamics described in terms of electric loops and the Higgs phase described in terms of Nielsen-Olesen magnetic vortices.

We can also ask whether the loops are no more than a useful representation or if, perhaps, they have a deeper physical meaning. Lattice QED exhibits a confining-deconfining transition, although in principle, ordinary continuum QED has only one non-confining

phase. However, there are studies which indicate that also there is a phase transition for QED in the continuum [30]. In addition to the usual weak coupling phase, a strong coupling confining phase exists above a critical coupling  $\alpha_c$ . This new phase could explain a mysterious collection of data from heavy ion collisions [31]-[35]. The unexpected feature is the observation of positron-electron resonances with narrow peak energy in the range of 1.4-1.8 MeV. This suggest the existence of 'electro-mesons' in a strongly coupled phase of QED. Moreover, a new two-phases model of continuum QED and a mass formula for taking account of the positronium spectrum in the strong coupling phase has been regarded recently [36]. Thus, in principle, we can speculate about the existence in nature of abelian electric tubes providing a real support for abelian loops and the possibility of being on equal footing with the observed magnetic vortices.

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## References

- [1] R.Gambini and A.Trias, Phys.Rev.D **22** (1980) 1380.
- [2] R.Gambini and A.Trias, Phys.Rev.D **23** (1981) 553.
- [3] A. Ashtekar Phys. Rev. Lett. **57**, (1986) 2244; A. Ashtekar, Phys. Rev. **D36**, (1987) 1587.
- [4] C. Rovelli and L. Smolin, Phys. Rev. Lett. **61** (1988) 1155; Nuc. Phys. **B331** (1990) 80.
- [5] R. Gambini Phys. Lett. **B235** (1991) 180;
- [6] R.Gambini and A.Trias, Nuc. Phys. **B278** (1986) 436.
- [7] R.Gambini, L.Leal and A.Trias, Phys.Rev.**D39** (1989) 3127.
- [8] C. Di Bartolo, R.Gambini and A.Trias, Phys.Rev.**D39** (1989) 1756.
- [9] J.M. Aroca and H. Fort, Phys.Lett.**B299** (1993) 305; J.M. Aroca and H. Fort, Phys.Lett.**B317** (1993) 604.
- [10] S. Mandelstam, Phys. Rev. **D19** (1979) 2391.
- [11] D.Armand Ugon, R.Gambini, J.Griego and L.Setaro, Preprint IF/FCN-93 July, 1993.
- [12] J.M.Aroca and H.Fort, Phys.Lett.**B325** (1994) 166.
- [13] A. M. Polyakov, Nuc. Phys. **B164**, (1979) 171.
- [14] J.L. Gervais and A. Neveu, Phys. Lett.**80B** (1979) 255.
- [15] H.B.Nielsen and P.Olesen, Nucl. Phys. **B61** (1973) 45.
- [16] M. Baig and E. Dagotto, Nucl. Phys. **B17** (Proc. Suppl.) (1990) 671; M. Baig, E. Dagotto and E. Moreo Phys. Lett. **242** (1990) 444.
- [17] A. H. Guth, Phys.Rev.**D21** (1980) 2291.
- [18] R.Savit, Rev.Mod.Phys.**52** (1980) 453.
- [19] A.K. Bukenov, U.J. Wiese, M.I.Polikarpov and A.V. Pochinskii , Phys.At.Nucl.**56** (1993) 122.
- [20] T. Banks, R. Myerson and J.B. Kogut, Nucl. Phys. **B129** (1977) 493.
- [21] D. Weingarten, Phys.Lett. **B90** (1980) 280.
- [22] V.A. Kazakov, T.A. Kozhamkulov and A.A. Migdal, Sov.J.Nucl.Phys. **43** (1986) 301.

- [23] J.M.Aroca and H. Fort Preprint UAB-FT-329/94.
- [24] M.I.Polikarpov U.J.Wiese and M.A.Zubkov, Phys.Lett. , **B309** (1993) 133.
- [25] J. Jerzak, T. neuhaus and P.M. Zerwas, Phys.Lett. **B133** (1983) 103.
- [26] V. Azcoiti, G. Di Carlo and A. F. Grillo, Phys. Lett. **B267** (1991) 101.
- [27] C.B. Lang and T. Neuhaus, Nuclear Physics **B** (Proc. Suppl.) **34** (1994) 543.
- [28] W. Kerler, C. Rebbi and A. Weber, Preprint BU-HEP 94-7.
- [29] M. Baig and H. Fort, Preprint UAB-FT 338/94 (Phys. Lett. B, in press)
- [30] T. Maskawa and H. Nakajima, Progr.Theor.Phys. **52** (1974) 1326; T. Maskawa and H. Nakajima, Progr.Theor.Phys.**54** (1975) 860; R.Fukuda and T.Kugo, Nucl.Phys.**B117** (1976) 250; V.A.Miransky, Nuovo Cimento **90A** (1985) 149; V.A.Miransky and P.I.Fomin, Sov.J.Part.Nucl.**16** (1985) 203;
- [31] J. Schweppe et al, Phys.Rev.Lett.**51** (1983) 2261.
- [32] M.Clemente et al, Phys.Lett.**B137** (1984) 41.
- [33] T. Cowan et al, Phys.Rev.Lett.**54** (1985) 1761; T. Cowan et al, Phys.Rev.Lett.**56** (1986) 444.
- [34] H. Tsertos et al, Phys.Lett.**B162** (1985) 273; Z.Phys.**A326** (1987) 235.
- [35] T. Cowan and J.Greenberg,in *Physics of Strong Fields* edited by W.Greiner (Plenum, New York, 1987).
- [36] Awada and Zoller, Nucl. Phys. **B365** 699 (1993).

## Figure Captions.

- 1a** Histogram of the plaquette energy density corresponding to 80.000 measures on a  $12^4$  lattice just at the Villain transition point  $\beta_V = 0.639$ .
- 1b** Time evolution of the total internal energy during the simulation. Each point is the average over 100 consecutive iterations.
- 2a** Typical loop configurations, obtained by intersecting the lattice with  $x_4 = 6$ , at  $\beta_V = 0.5$  (strong coupling phase).
- 2b** Typical loop configurations, obtained by intersecting the lattice with  $x_4 = 6$ , at  $\beta_V = 1$  (weak coupling phase).

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